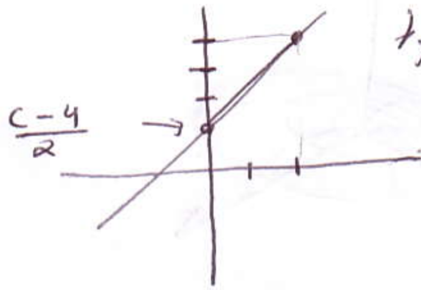
Solutions to HW#5

1. a) $f(x,y) = 4-3x+2y$, $f(x,y)=c$

$$4-3x+2y=c \Rightarrow 2y = 3x+c-4$$

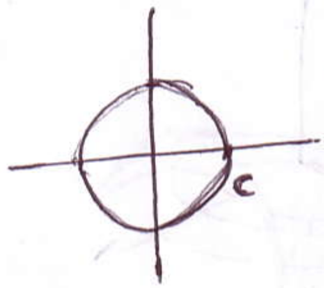
$$\Rightarrow y = \frac{3}{2}x + \frac{c-4}{2}$$



typical level curve.

b) $f(x,y) = (x^2+y^2)^{1/2}$, $f(x,y)=c$

$$x^2+y^2=c^2$$

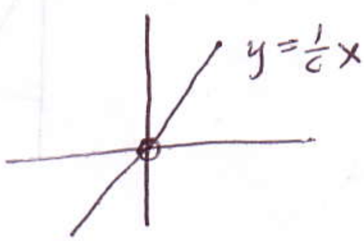


typical level curve.

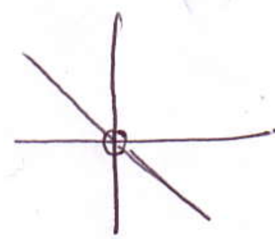
c) $f(x,y) = \frac{x}{y}$, $f(x,y)=c$

$$\frac{x}{y}=c \Rightarrow \frac{y}{x}=\frac{1}{c} \quad y=\frac{1}{c}x$$

if $c>0$



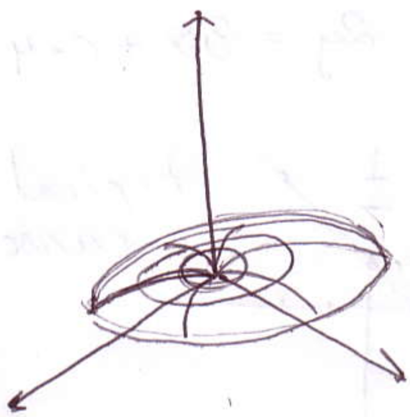
if $c<0$

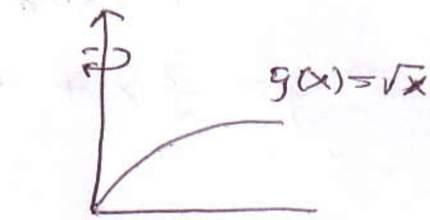


if $c=0$

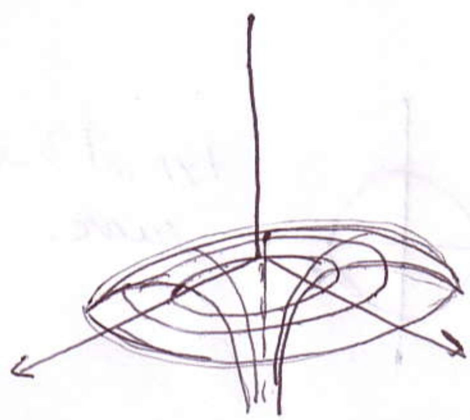
2. a) $f(x,y) = (x^2+y^2)^{1/4} = \left[(x^2+y^2)^{1/2}\right]^{1/2}$

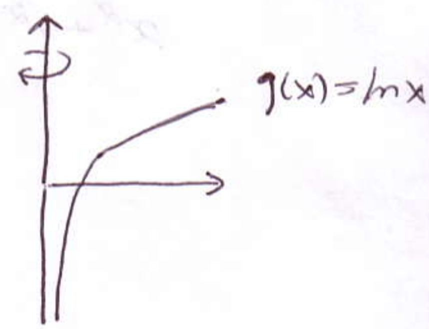
In other words, f is the surface of the curve obtained by rotating $g(x)=\sqrt{x}$ about the z-axis



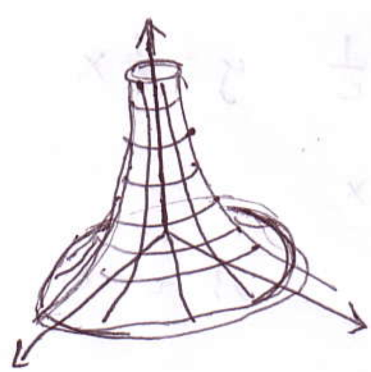


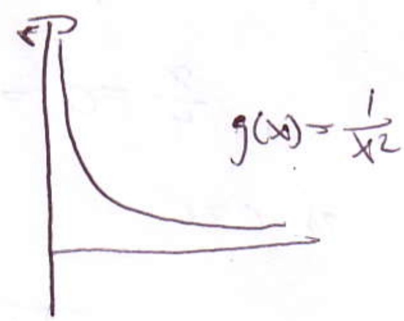
b) $\ln\left[(x^2+y^2)^{1/2}\right]$



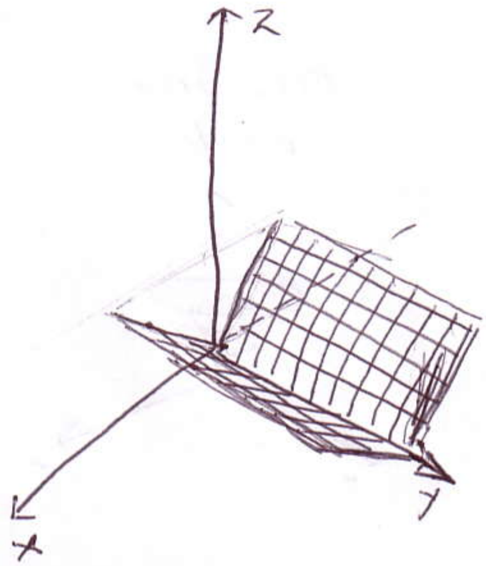


c)

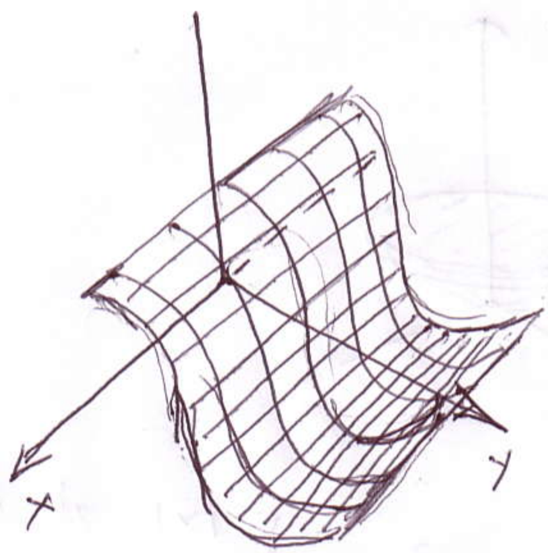




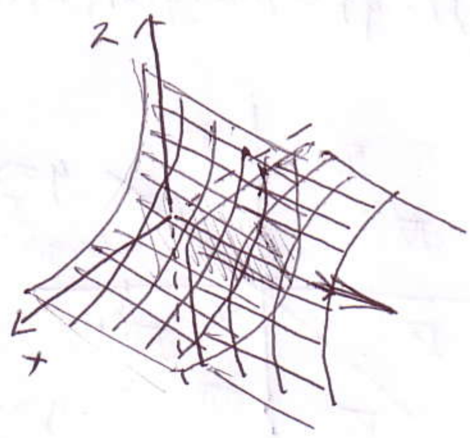
3. a) $f(x,y) = |x|$



b) $f(x,y) = \cos y$



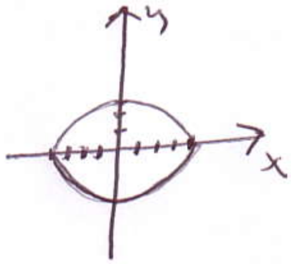
c) $f(x,y) = \frac{1}{x}$



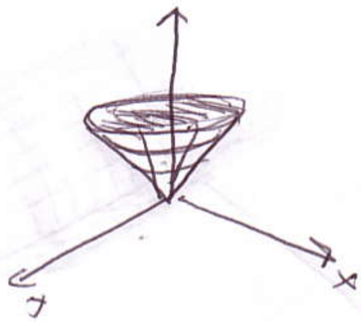
4. a) $f(x,y) = \sqrt{\frac{x^2}{16} + \frac{y^2}{9}}$

Its graph is an elliptic cone

view from above

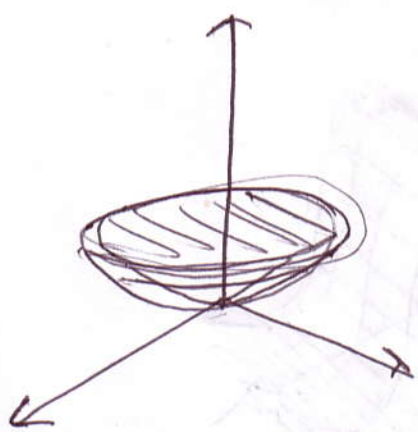


view from a side

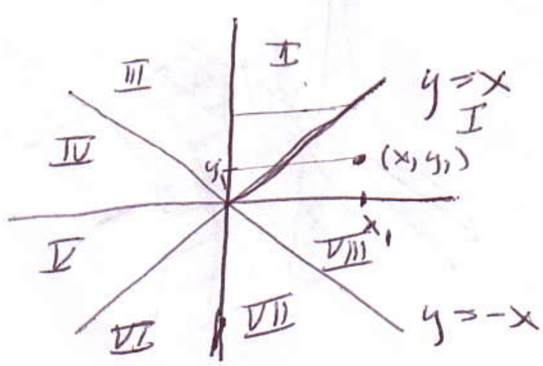


b) $f(x,y) = \frac{x^2}{9} + \frac{y^2}{16}$

Its graph is an elliptic paraboloid



5. a) $f(x,y) = \max\{|x|, |y|\} = \begin{cases} |x| & \text{if } |x| \geq |y| \\ |y| & \text{if } |y| \geq |x| \end{cases}$



$x_1 \geq y_1 > 0$

For any point (x,y) in region I $|x| > |y|$

Similarly for any (x,y) in region VIII $|x| > |y|$

In regions II & III $|y| > |x|$

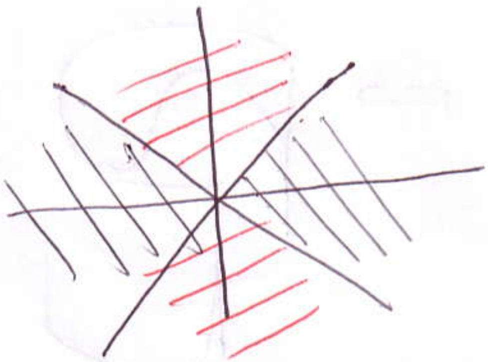
In regions IV & V $|x| > |y|$

And in regions VI & VII $|y| > |x|$

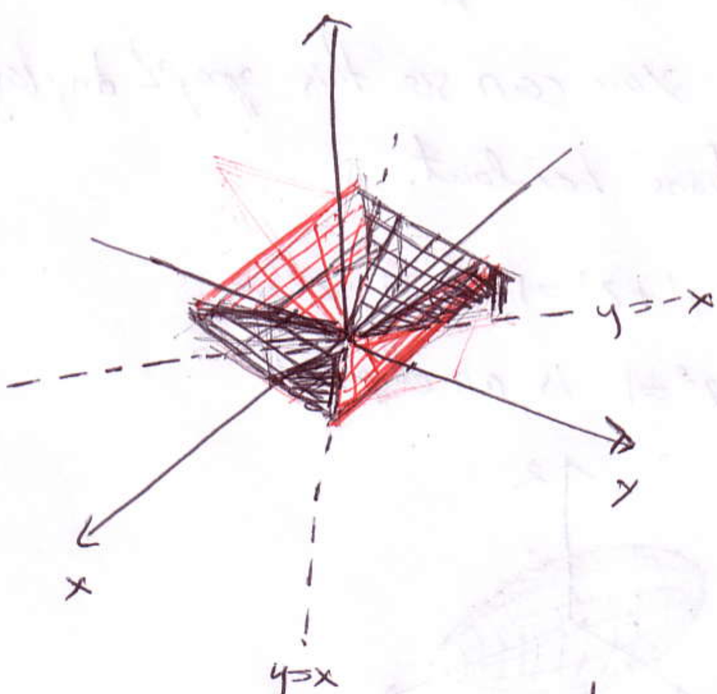
Hence

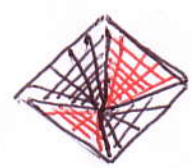
— $|y| > |x|$

— $|x| > |y|$



The graph is as follows





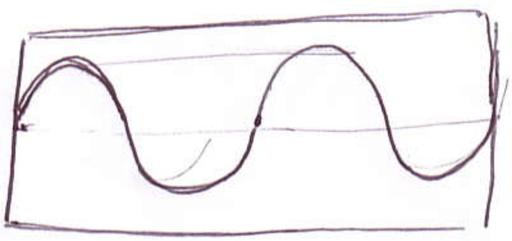
It will look like an inverted pyramid.

b) $f(x,y) = \frac{2xy}{x^2+y^2}$

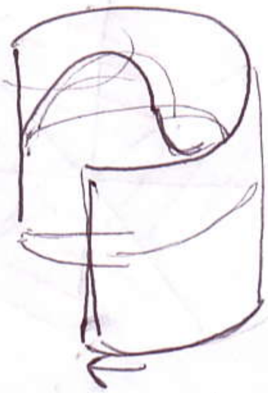
Observe that $f(r\cos\theta, r\sin\theta) = \frac{2r^2\sin\theta\cos\theta}{r^2} = \sin 2\theta.$

It follows that the graph of f would be generated by a "smoke ring" expanding outward from the z-axis.

This smoke ring is obtained by taking the planar graph of $z = \sin 2\theta$ $\theta \in [0, 2\pi]$ and rolling it into a cylinder.



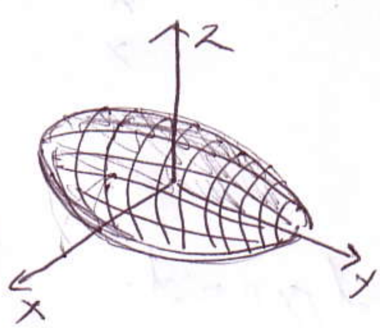
$\longmapsto$



This graph will be similar in appearance to the graph of $g(x,y) = \frac{x^2-y^2}{x^2+y^2}$. You can see this graph displayed in the "graphs of functions" handout.

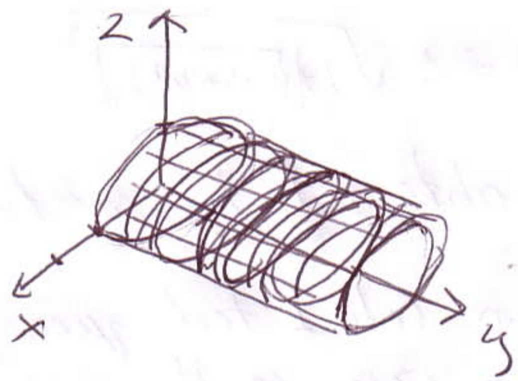
6. a) Since $x^2+y^2+z^2=1$ is a sphere

$\frac{x^2}{4} + \frac{y^2}{9} + z^2 = 1$ is an ellipsoid



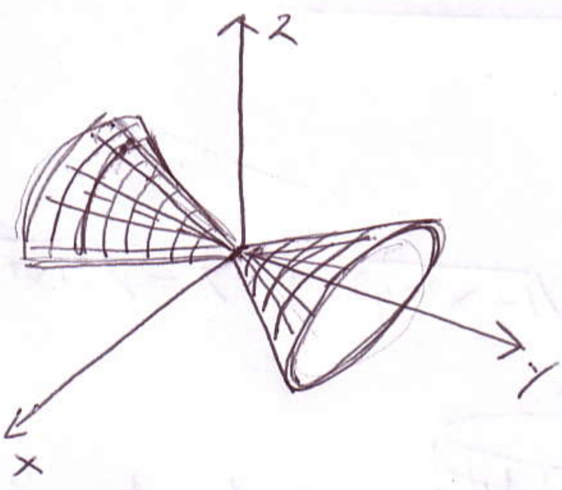
b) $\frac{x^2}{4} + z^2 = 25 \iff \frac{x^2}{100} + \frac{z^2}{25} = 1$

the graph of this surface is an elliptic cylinder



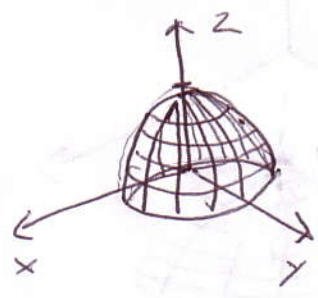
c) $x^2 - y^2 + 16z^2 = 0 \iff x^2 + 16z^2 = y^2$

$\iff \pm\sqrt{x^2+16z^2} = y$ the graph of this surface is an elliptic (mathematical) cone.



d) $x^2+y^2+z-1=0 \iff z = 1-(x^2+y^2) = 1-\left(\sqrt{x^2+y^2}\right)^2$

the graph is therefore



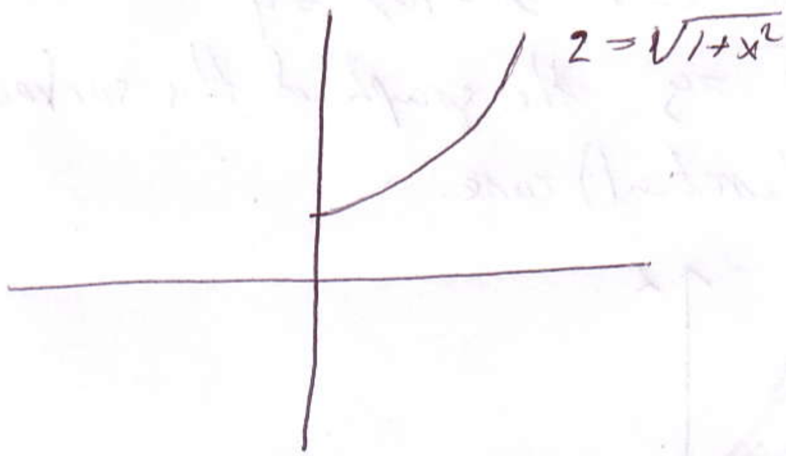
7. to analyze $-\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ consider

$-x^2-y^2+z^2=1$ instead

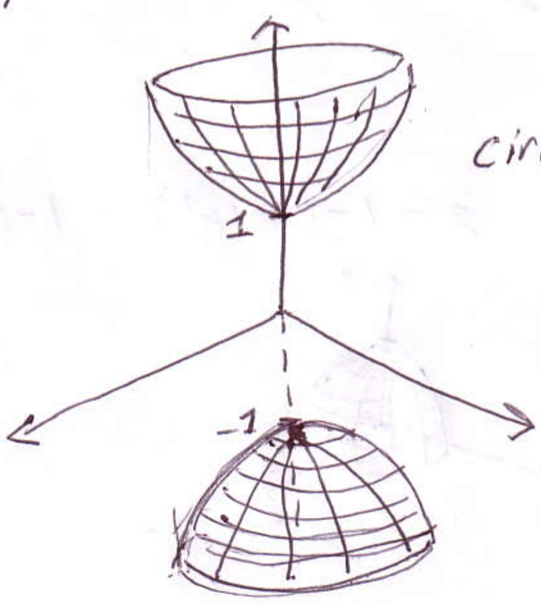
$z^2 = 1+x^2+y^2 \Rightarrow z=\pm\sqrt{1+[\sqrt{(x^2+y^2)}]^2}$ where

$z=\sqrt{1+[\sqrt{x^2+y^2}]^2}$ is obtained by rotating $z=\sqrt{1+x^2}$

about the z-axis. From Calc I techniques the graph of $z=\sqrt{1+x^2}$, $x \geq 0$ in the x-z plane is

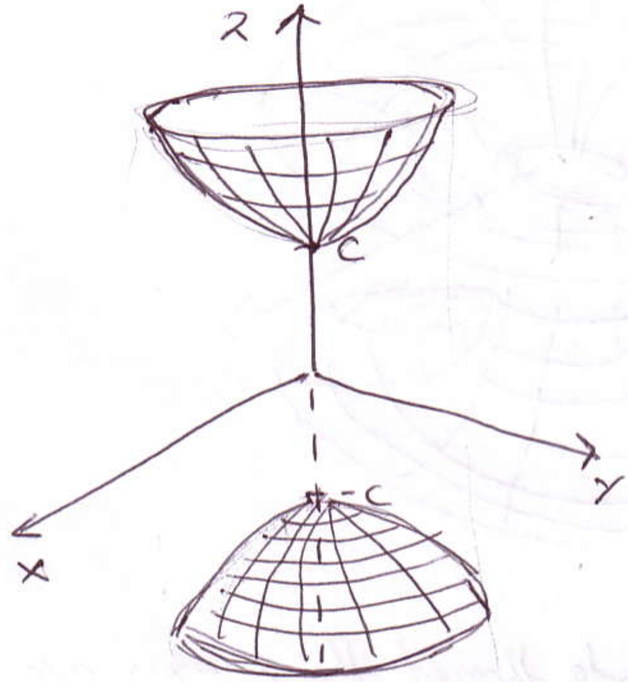


Thus the graphs of $z=\sqrt{1+x^2+y^2}$ & $z=-\sqrt{1+x^2+y^2}$ are



circular hyperboloid

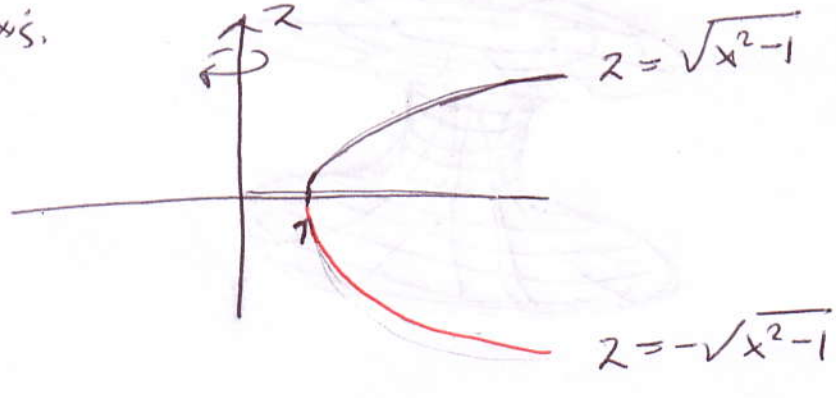
Since $-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ is obtained from $-x^2 - y^2 + z^2 = 1$ by stretching the x, y, and z axes by a factor of a, b, and c respectively, its graph is an elliptic hyperboloid.



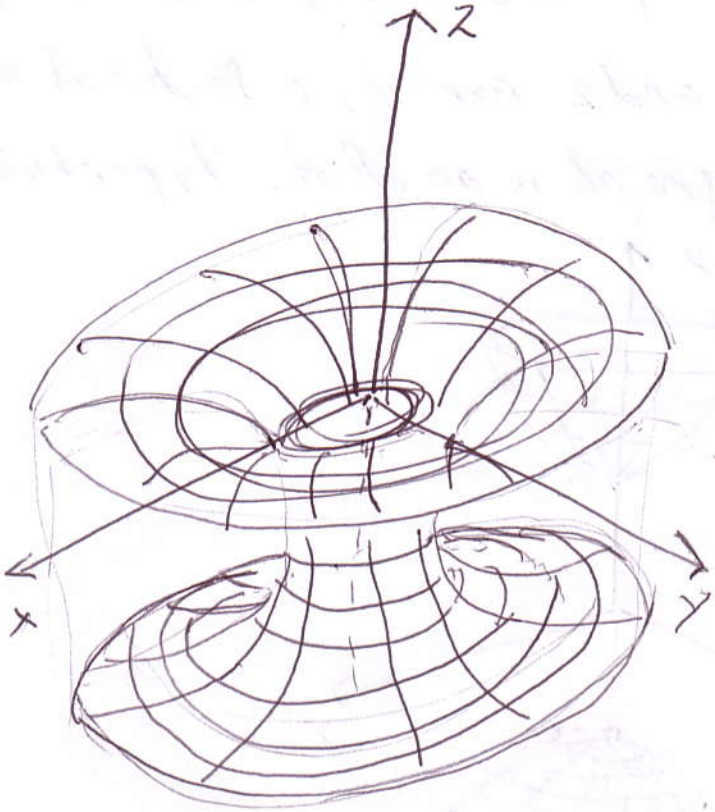
8. $\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ can be obtained from $x^2 + y^2 - z^2 = 1$

$\pm\sqrt{x^2+y^2-1} = z$ which is obtained from spinning $z = \pm\sqrt{x^2-1}$, $x \geq 1$

about the z-axis.



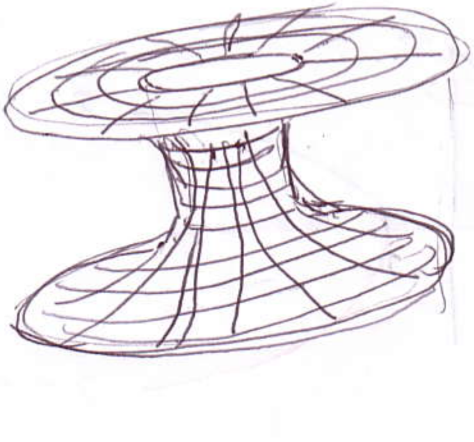
Hence the graph is



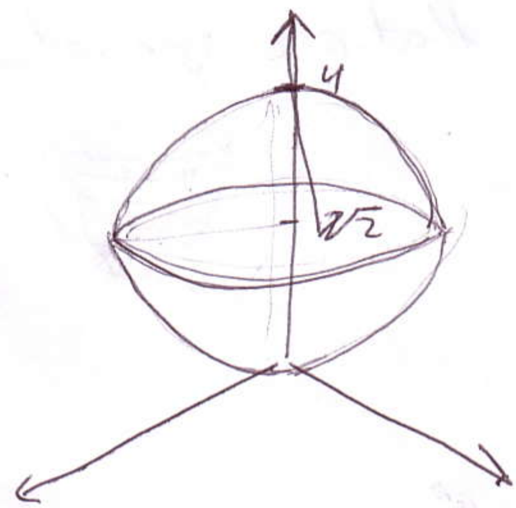
Its cross sections ~~thro~~ through the z-axis are circles.

This worm-hole-looking structure is stretched into an elliptical two hole funnel when $x^2+y^2-z^2=1$ is transformed into

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$$



9. a)

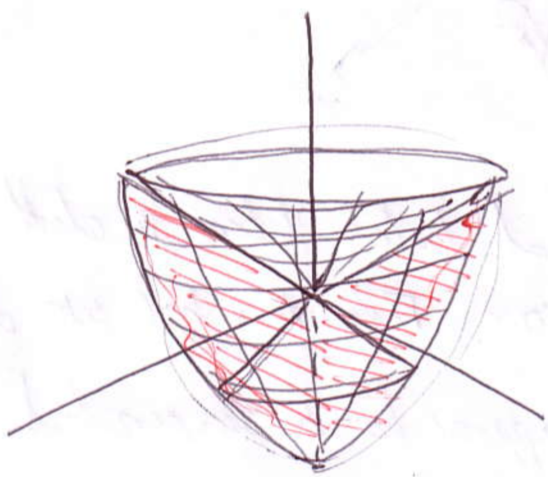


$z = x^2 + y^2 = 4 - x^2 - y^2$ when $x^2 + y^2 = 2$

The region is the space enclosed between two bowls

Remark: This region is NOT a ball.

b)



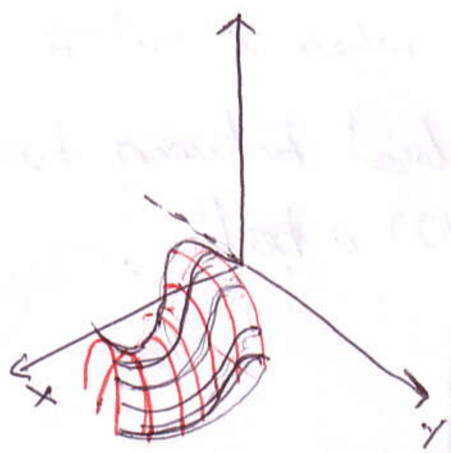
$z^2 = x^2 + y^2 \qquad z = x^2 + y^2 - 3$

$z = z^2 - 3$

$z^2 - z - 3 = 0 \qquad z = \frac{1 \pm \sqrt{1+12}}{2} = \frac{1 \pm \sqrt{13}}{2}$

There are two solids that are generated:

10. a) $f(x,y) = \sin x - y^2$



The picture generated looks like a cloth-hanger-bar bent in the shape of $\sin x$. From this bar we drop the parabulas $-y^2$. These parabulas appear to be suspended like cloth hangers on the sinusoidal bar.

b) $f(x,y) = \frac{1}{x} + y$